Birzeit University
Mathematics Department
Math 235 - First Exam
First Summer 2012

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Question 1. *(20%) Decide whether the following statements are* ***TRUE*** *or* ***FALSE***

1. True If $R(x)$ is the revenue function, then we find all possible points where $R(x)$ could be maximized by solving $MR(x) = 0$.

2. False The point of intersection of the supply and demand functions is called break even point.

3. True If $f''(0) = 0$, then $f(x)$ has a point of inflection at $x = 0$.

4. False $f'(-3) = 0$ and $f'(x)$ changes from positive on the left to negative on the right of $x = -3$, then $f(-3)$ is relative minimum

   f' + − ; −3

5. False If $f'(1) = 7$, then $f(x)$ is increasing at $x = 1$.

6. False If $f(x) = 7 + 3x - x^3$, $f'(x) = 3 - 3x^2$, then the critical value is $x = 1$.

   $3 - 3x^2 = 1$ $3 = 3x^2$ $x^2 = 1$

7. True The typical linear supply function have positive slope.

8. false If $f'(-2) = 0$, then a relative maximum or minimum occurs at $x = -2$.

9. False If $f(x) = (3x^4 + 1)^{10}$, then $f' = 10(3x^4 + 1)^9(4x^3 + 1)$.

   $12x^3$

10. True If the supply is given by $p = 0.1x + 20$, and the demand is $p = 130 - 0.1x$, then the equilibrium point is $(550 units, \$75)$.

$.1x + 20 = 130 - .1x$

$-130 + .1x$

$.2x - 110 = 0$

$.2x - 110 = 0$

$.2x = 110$

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Question 2. (36%).Circle the most correct answer:

1. If \$2500 investment grows to \$2875 in 15 months. What simple interest rate was earned

    (a) %12
    (b) %14
    (c) %10
    (d) %8

~~2500~~

$I = Prt$

$\boxed{FV = P + I}$

$2875 = 2500 + I$

$375 = 2500 \times r \times \frac{15}{12}$

$12 \times .15 = \frac{15}{12} r$

2. If the profit function is $P(x) = -0.2x^3 + 3x^2 + 6$, then the point of diminishing returns for the profit is at

    (a) $x = 6$
    (b) $x = 4$
    (c) $x = 5$
    (d) $x = 3$

$P' = -.6x^2 + 6x$

$P'' = -1.2x + 6$

$-1.2x + 6 = 0$

$1.2x = 6$

3. The critical points of $y = (x+2)^{\frac{2}{3}}$ are at

    (a) $x = -2$
    (b) $x = 0$
    (c) $x = \frac{2}{3}$
    (d) None of the above.

$\frac{2}{3}(x+2)^{-\frac{1}{3}} \times 1$

$\frac{2}{3}(x+2)^{\frac{1}{3}} = 0$

4. If the average cost function is $\overline{C} = 2800 + \frac{298600}{x}$, then the cost function is

    (a) $C(x) = 2800 + 298600x$
    (b) $C(x) = \frac{2800}{x} + \frac{298600}{x^2}$
    (c) $C(x) = 298600x + 2800$
    (d) $C(x) = 2800x + 298600$

$\overline{C} = \frac{R}{x}$

$R = PX$

5. If the revenue function is $R(x) = 15(2x+1)^{-1} + 20x - 15$, What is the marginal revenue

(a) $30(2x+1)^{-2} + 20$
(b) $\frac{-30}{2x+1} + 20$
(c) $\frac{-30}{(2x+1)^2} + 20x$
(d) $\frac{-30}{(2x+1)^2} + 20$

$-1 \times 15(2x+1)^{-2} \cdot 2 + 20$

$= -15(2x+1)^{-2} \cdot 2 + 20$

-30

6. The supply function for a product is $p = 5x + 1500$, and the demand function is $p = -3x + 3100$. If the wholesaler is taxed \$16, then the equilibrium point is

(a) $(198, 2490)$
(b) $(200, 2500)$
(c) $(198, 2506)$
(d) $(202, 2494)$

$5 + 16$

$5x + 1500 + 16 = -3x + 3100$

$5x + 1516 = -3x + 3100$

$+3x$

$8x + 1516 = 3100$

$8x = 1584$

7. What is the instantaneous rate of change of the marginal profit if the profit function is $P(x) = 120x - x^2$ when 40 units are produced and sold

(a) -2
(b) 40
(c) 3200
(d) None of the above

$P'(x) = 120 - 2x$

$P''(x) = -2$

8. How long does it take an investment to double if it is invested at %8 compounded annually

(a) $10 years$
(b) $8 years$
(c) $9 years$
(d) $11 years$

$fv = P\left(1+\frac{r}{m}\right)^{m \cdot t}$

$2P = P\left(1+\frac{.08}{1}\right)^{1 \cdot t}$

$\frac{2P}{P} = \frac{P}{P}(1.08)^{1 \cdot t}$

$2 = 1.08^{t}$

$\ln 2 = t \times \ln 1.08$

$.693 = t \times .076$

$-1+8=7$

$1-2+8=$

9. Find the absolute maximum of $f(x) = x^2 - 2x + 8$ over $[0, 3]$

(a) 11

(b) 8

(c) 7

(d) 3

$f'(x) = \boxed{2x-2}$

$2x - 2 = 0$

$2x = 2$

$x = 1$

$f(1) = 7 \quad f(0) = 8$

$f(3) = 9 - 6 + 8$

$3 + 8$

$= 11$

10. The function $f(x) = \frac{1}{3}x^3 - 2x^2 + 2x$ is concave up over

(a) $(-\infty, 2)$

(b) $(-2, \infty)$

(c) $(-\infty, -2)$

(d) $(2, \infty)$

$f' = x^2 - 4x + 2$

$f''(x) = 2x - 4$

$2x - 4 = 0$

$2x = 4$

$x = 2$

$\boxed{2x-4}$

11. If $y = (x^2+1)\sqrt{2x^3+1} + 2\pi^3$, find y'

(a) $\frac{3x^2(x^2+1)}{\sqrt{2x^3+1}} + 2x\sqrt{2x^3+1} + 6\pi^2$

(b) $\frac{3x^2(x^2+1)}{\sqrt{2x^3+1}} + 2x\sqrt{2x^3+1}$

(c) $\frac{3(x^2+1)}{2\sqrt{2x^3+1}} + (2x+1)\sqrt{2x^3+1}$

(d) $\frac{3x^2(x^2+1)}{2\sqrt{2x^3+1}} + 2x\sqrt{2x^3+1}$

$(x^2+1)(2x^3+1)^{-\frac{1}{2}} \cdot 6x^2) + (2x^3+1)^{\frac{1}{2}} \cdot 2x$

$(x^2+1)\left(\frac{1}{2\sqrt{2x^3+1}}\right)(6x^2)$

12. If the cost function is $C(x) = 3600 + 25x + \frac{1}{2}x^2$, and the revenue function is $R(x) = 175x - \frac{1}{2}x^2$. Then the break even occurs at

(a) $x = 70,\ x = 80$

(b) $x = 120,\ x = 30$

(c) $x = 100,\ x = 50$

(d) $x = 40,\ x = 110$

$3600 + 25x + \frac{1}{2}x^2 = 175x - \frac{1}{2}x^2$

$3600 - \boxed{150x} + x^2 = 0$

$x^2 - \boxed{150}x + 3600 = 0$

$(x - 120)(x + 30)$

30

Question 3. *(9%) Find the following limits:*

1. $\lim_{x\to-\infty} \frac{2x^3-4x^6+3}{2x^6+3x+4}$ $= \frac{-4}{2} = \boxed{2}$

$$\lim_{x\to-\infty} = \frac{2(-\infty)^3 - 4(-\infty)^6 + 3}{2(\infty-)^6 + 3(-\infty) + 4}$$

$$\frac{2(-\infty) - 4\times\infty + 3}{2(\infty) + 3(-\infty) + 4} = \frac{-2\infty - 4\infty + 3}{2\infty + 3\infty + 4}$$

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2. $\lim_{x\to3} \frac{x^3-3x^2}{x^2+x-12}$ $(x^2 + x - 12)$ x

$$\lim_{x\to 3} \frac{x^2(x-3)}{(x-3)(x+4)}$$

$$\lim_{x\to 3} \frac{x^2}{x+4} = \frac{(3)^2}{3+4} = \frac{9}{7} = \boxed{1.285}$$

3

3. $\lim_{x\to2} \frac{x^2-4}{2-x}$

$$\lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)}$$

$$\lim_{x\to 2} x+2 = \boxed{4}$$

2

7

$y(1.7) = (1.7)^3 - 9(1.7) - 2 = -12.38$ $(1.7, -2.38)$

$y(-1.7) = -(1.7)^3 - 9(-1.7) - 2 = 8.337$ $(-1.7, 8.337)$

Question 4. *(10%)*

1. *What is the equation of the tangent of* $f(x) = x^3 - 9x - 2$ *at* $x = 3$

ميل $f'(x) = 3x^2 - 9$

$= 3(3)^2 - 9$

$3 \times 9 - 9$

ميل $f'(x) = 18$

$f(3) = (3)^3 - 9 \cdot 3 - 2$

$f(3) = -2$

نقطة (x_1, y_1) $(3, -2)$, ميل 18

ميل

$y - y_1 = \text{slope}(x - x_1)$

$y + 2 = 18(x - 3)$

2. *At what point(s) does* $f(x) = x^3 - 9x - 2$ *have a horizontal tangent(s)*

$f'(x) = 0$

$f'(x) = 3x^2 - 9$

$3x^2 - 9 = 0$

$x = \sqrt{3} \pm$

$x = 1.7$

$x = -1.7$

point

$y(1.7) = (3)^3 - (9 \cdot 3) - 2 = -2$

$y(-3) = -27 + 27 - 2 = -2$

$3x^2 - 9 = 0$

$3x^2 = 9$ $x^2 = 3$

Question 5. *(5%) A company sales (in thousands of dollars) are*

$$\boxed{S(t) = 2t + \frac{1}{3}t^{\frac{2}{3}}}$$

$\frac{2}{3} - \frac{3}{1}$

where t is the time in months

1. *Find* $S'(t)$ $= 2 + \frac{2}{3} \cdot \frac{1}{3} t^{-\frac{1}{3}}$

$= 2 + \frac{2}{9} t^{-\frac{1}{3}}$

$= 2 + \frac{2}{9}$

2. *Estimate the total sales in the first 10 months*

$S(10) = S'(9) + S(9)$

$S'(9) = 2 + \frac{2}{9\sqrt[3]{9}}$

$=$

$= 2 + \frac{2}{9 \times 2.08}$

$= 2 + \frac{2}{18.72}$

1/2

Question 6. *(20%) For a certain company producing* 400 *units costs* \$6000, *and producing* 500 *units costs* \$7000. *If the price demand function is* $p = 60 - 0.2x$ *answer the following*

1. *Find the total cost function.*

$\cancel{7000 - 6000} = \frac{y_2 - y_1}{x_2 - x_1}$ unit $= \frac{7000 - 6000}{500 - 400} = \frac{1000}{100} = 10$

$C(x) = a + b(x)$
$C(x) = a + 10(x)$
$6000 = a + 10(400)$
$6000 = a + 4000$
$-4000 \quad -4000$
$= a = 2000$

$\boxed{C(x) = 2000 + 10x}$

2. *Find the total revenue function.*

total revenue $= px$
$= (60 - .2x)x$
$60x - .2x^2$

3. *Find the total profit function.*

profit = revenue − cost
$60x - .2x^2 - (2000 + 10x)$
$\cancel{16x - .2x^2 - 2000}$
$P(x) = \cancel{-.2x^2 - 4x - 2000}$

$P(x) = 60x - .2x^2 - 2000 - 10x$
$-.2x^2 + 50x - 2000$

4. *What is the maximum profit.*

$P'(x) = \cancel{-.4x - 4}$ $\quad P(-10) = \cancel{.2(-10)^2 - 4(-10) - 2000}$
$\cancel{.4x - 4 = 0}$ $\quad = .2 \cdot 100 + 40 - 200$
$\cancel{-.4x = 4}$ $\quad 20 + 40 - 200$
$\cancel{\frac{-.4x}{.4} = \frac{4}{.4}}$ $\quad -440$
$\cancel{-x = 10}$
$\cancel{x = -10}$

$\boxed{P'(x) = .4x + 50}$
$.4x + 50 = 0$
$\frac{-.4x}{.4} = \frac{-50}{.4}$
$\boxed{x = 125}$

5. *What price will maximize the profit.*

~~total P X~~

$\cancel{P(-10)}$ $p(125) = 60 - .2x$
$= 60 - .2(125)$
$= 60 - 25$

$P(125) = -.2(125)^2 + 50(125)$
-2000